

# An Introduction to Modern Panel Data Methods: Differences-in-Differences and Synthetic Control

Diego Ciccia

Kellogg School of Management, Northwestern University

# Introduction

- This set of lectures revolves around three causal inference methods for panel data
  - Differences-in-Differences (DID)
  - Synthetic Control (SC)
  - Synthetic Differences-in-Differences (SDID)
- The main focus is on SDID, hence DID and SC only serve as a stepping stone
- These methods work within similar frameworks
  - I will try to strike the *similarities-in-similarities*
  - I will discuss DID in a way that is conducive to SC and SDID
- Objective: give you a user-oriented overview of DID/SC/SDID
  - Broad enough to understand what the packages are doing
  - Not *too* simplified, to give you solid grounds to read the papers/books if needed

# Table of Contents

- 1 Introduction
- 2 Potential outcome framework
- 3 Treatment design with panel data
- 4 Model-based framework and latent factor model
- 5 Differences-in-Differences
- 6 Synthetic Control
- 7 Application: effect of tobacco control in California (Abadie et al., 2010)
- 8 Outro

# Sources

- Clément de Chaisemartin and Xavier D'Haultfoeuille, *Causal Inference with Differences-in-Differences: Credible Answers to Hard Questions*, Online version
- Kirill Borusyak, *Applied Econometrics Notes*, UC Berkeley, Github
- Ivan Canay, *480-3 Econometrics*, Northwestern University, Slides
- (New book!) Damian Clarke. 2026. *Applied Microeconometrics*. MIT Press

## Static potential outcome framework, I

- In impact evaluation, intervention units are assigned to some treatment  $D$ 
  - Upskilling-reskilling training program, local/regional policies, ...
- Later on, data on some outcome  $Y$  is collected
- Treatment  $D$  arguably has some effect on  $Y$ , which is unknown to the researchers
- A way to formalize treatment effects is via the **potential outcome framework**
  - Neyman proposed this framework in his master's thesis in 1923
- The potential outcome for a unit assigned to treatment  $d$  is  $Y(d)$ 
  - If  $D \in \{0, 1\}$ ,  $Y(1)$  outcome had the unit been treated,  $Y(0)$  untreated
- The (individual) treatment effect (TE) is

$$TE \equiv Y(1) - Y(0)$$

## Static potential outcome framework, II

- People generally *respond* differently to treatment
  - Some people may learn a lot from a job training program, other way less
  - As such, TE is often formalized as a random variable
  - We are interested in some characteristics of its distribution, e.g., mean, variance (!)
- Since policies are generally thought to be scaled-up to the whole population, we are often interested in *average* treatment effect

$$ATE \equiv \mathbb{E}[Y(1) - Y(0)]$$

- Unless we have experimental data, most of the time we can at most identify the average treatment effect among people *who were treated*

$$ATT \equiv \mathbb{E}[Y(1) - Y(0) | D = 1]$$

## Static potential outcome framework, III

- **Fundamental problem of causal inference:** either  $Y(1)$  or  $Y(0)$  is observed
- Our data generally consists of  $(Y, D)$ , where

$$\begin{aligned} Y &= Y(D) = Y(1)D + Y(0)(1 - D) \\ &= Y(0) + D \cdot TE \end{aligned}$$

- Data  $(Y, D)$  does not reveal  $TE = Y(1) - Y(0)$ 
  - $Y = Y(1)$  if  $D = 1$ , we observe treated potential outcomes among treated units
  - $Y = Y(0)$  if  $D = 0$ , we observe untreated potential outcomes among untreated units
  - No one has  $D = 1$  and  $D = 0$  simultaneously
  - $Y(0)$  if  $D = 1$  and  $Y(1)$  if  $D = 0$  are unknown
- **Goal:** under which assumptions, characteristics of TE are **identified** by the data?
  - Identified: expressed as a function of characteristics of **observable** data  $(Y, D)$

# The meaning of ATT

- In DID/SC/SDID, the target parameter is ATT
  - ATT conditions on the subpopulation that was treated
  - Outside of randomized trials, this subpopulation may self-select into treatment
- Another useful interpretation of ATT:

$$\mathbb{E}[Y - Y(0)] = ATT \cdot P(D = 1)$$

- Keep realized outcomes as they are → status-quo policy
- Consider a policy where no one is treated → counterfactual policy
- The *sign* of ATT tells us whether *on average* units are better off keeping their current treatment status vs being all untreated

# Panel data

- So far, I have described the potential outcome framework for some undefined population, abstracting from a particular characterization of the data
- DID/SC/SDID require access to **panel** data:
  - Units  $\{1, \dots, G\}$  are each observed in periods  $\{1, \dots, T\}$
- As a result, the effective data at our disposal is

$$\{(Y_{g,t}, D_{g,t})\}_{(g,t) \in \{1, \dots, G\} \times \{1, \dots, T\}}$$

- Double-indexing allows us to rewrite the collection of outcomes and treatments in a convenient *matrix* form

# Treatment design, I

- Each unit  $g$  can either be treated ( $D_{g,t} = 1$ ) or untreated ( $D_{g,t} = 0$ ) at time  $t$
- Across all  $t$ , unit  $g$  has treatment vector  $\mathbf{D}_g = (D_{g,1}, \dots, D_{g,T})$
- The **design** of a study is the collection of treatment vectors

$$\mathbf{D} = (\mathbf{D}_1, \dots, \mathbf{D}_G) = \begin{pmatrix} (g, t) & 1 & 2 & \dots & T \\ 1 & D_{1,1} & D_{1,2} & \dots & D_{1,T} \\ 2 & D_{2,1} & D_{2,2} & \dots & D_{2,T} \\ \vdots & \vdots & \ddots & \dots & \vdots \\ G & D_{G,1} & D_{G,2} & \dots & D_{G,T} \end{pmatrix}$$

- $\mathbf{D}$  is  $G \times T$  matrix whose  $(g, t)$  entry is treatment status  $D_{g,t}$

## Treatment design, II

- Basic design:
  - Treated units get treated at the **same** time and never go back to untreated
  - Ex. units 1 and 2 get treated at 3, while unit 3 is a **never-treated** unit

$$\mathbf{D} = \begin{pmatrix} (g, t) & 1 & 2 & 3 & 4 \\ 1 & 0 & 0 & 1 & 1 \\ 2 & 0 & 0 & 1 & 1 \\ 3 & 0 & 0 & 0 & 0 \end{pmatrix}$$

- Staggered design:
  - Treated units get treated at a **different** time and never go back to untreated
  - Ex. units 1 and 2 are in different **cohorts**

$$\mathbf{D} = \begin{pmatrix} (g, t) & 1 & 2 & 3 & 4 \\ 1 & 0 & 0 & 1 & 1 \\ 2 & 0 & 0 & 0 & 1 \\ 3 & 0 & 0 & 0 & 0 \end{pmatrix}$$

## Treatment design, III

- For now, I only consider basic designs
  - I will cover staggered designs in the context of SDID
  - One can also have more general settings (e.g., on-and-off)
- Basic designs clearly *partition* units and periods in two subgroups each:
  - Units in  $\{1, \dots, G\}$  either belong to  $G_1$  (treated group) or  $G_0$  (untreated)
  - Periods in  $\{1, \dots, T\}$  either belong to  $T_1$  (post) or  $T_0$  (pre)
- I use  $|S|$  to refer to how many units are in set  $S$ 
  - This is generally referred to as the *cardinality* of set  $S$
  - Why? It makes the notation for the estimators nicer, also avoids tedious algebra
- With this notation, we have
  - $|G_1|$  units that are treated for  $|T_1|$  periods
  - Treatment starts at period  $|T_0| + 1$
  - $|G_0|$  units that are never-treated

## Model-based perspective on inference

- A non-trivial point in this type of studies is: **what is random in the data?**
  - Units sampled from a larger population? Treatment assignment?
  - The answer to this question affects the way we draw inference on estimators
  - Section 2.4 of *Credible Answers to Hard Questions* really elucidates on this point
- The framework that is generally used in DID/SC/SDID is **model-based**
  - Treatment design  $\mathbf{D}$  is fixed (non-stochastic)
  - Randomness comes from shocks that affect units' outcomes *on top of their treatment*
- This framework is generally enforced by **conditioning on the design**
  - Every expectation  $\mathbb{E}[\cdot]$  is implicitly  $\mathbb{E}[\cdot|\mathbf{D}]$
  - Since  $(G_0, G_1, T_0, T_1)$  are functions of  $\mathbf{D}$ , they can be *taken out* of the expectations
  - What remains random conditioning on  $\mathbf{D}$  is *potential outcomes*

## Potential outcomes in panel data, I

- At most, potential outcome of unit  $g$  at time  $t$  depends on  $g$ ,  $t$  and the *design*

$$Y_{g,t} = Y_{g,t}(\mathbf{D})$$

- Dependence of potential outcomes on  $(g, t)$  mirrors the model-based framework
  - $(g, t)$ -level shocks affect potential outcomes *even* conditioning on treatment
- We make a few assumptions:
  - **No interference:** Potential outcomes of unit  $g$  only depend on unit  $g$ 's treatment

$$Y_{g,t} = Y_{g,t}(\mathbf{D}_g) = Y_{g,t}(D_{g,1}, \dots, D_{g,T})$$

- **No anticipation:** Potential outcomes at time  $t$  do not depend on future treatment

$$Y_{g,t} = Y_{g,t}(D_{g,1}, \dots, D_{g,t})$$

## Potential outcomes in panel data, II

- In SC/SDID, the following assumption is also maintained
  - **No dynamics:** Potential outcomes at time  $t$  do not depend on past treatment

$$Y_{g,t} = Y_{g,t}(D_{g,t})$$

- No dynamics rules out compounded effects from persistence of treatment status
  - For instance, it assumes that just-laid-off workers are on the same treatment trajectory as workers who have been out of the workforce for a long time
- This assumption is often relaxed in some DID applications (e.g., event-study)
  - I keep it to understand the differences in DID/SC/SDID on an equal footing

## Potential outcomes in panel data, III

- Under no dynamics, one can collect all observed outcomes in matrix form
- Say we have design

$$\mathbf{D} = \begin{pmatrix} (g, t) & 1 & 2 & 3 \\ 1 & 0 & 1 & 1 \\ 2 & 0 & 1 & 1 \\ 3 & 0 & 0 & 0 \end{pmatrix}$$

- The observed outcomes are

$$\mathbf{Y} = \begin{pmatrix} Y_{1,1}(0) & Y_{1,2}(1) & Y_{1,3}(1) \\ Y_{2,1}(0) & Y_{2,2}(1) & Y_{2,3}(1) \\ Y_{3,1}(0) & Y_{3,2}(0) & Y_{3,3}(0) \end{pmatrix}$$

- We do not observe  $Y_{g,t}(0)$  for entries in red

## Potential outcomes in panel data, IV

- By just adding and subtracting, we get

$$\mathbf{Y} = \begin{pmatrix} Y_{1,1}(0) & Y_{1,2}(1) & Y_{1,3}(1) \\ Y_{2,1}(0) & Y_{2,2}(1) & Y_{2,3}(1) \\ Y_{3,1}(0) & Y_{3,2}(0) & Y_{3,3}(0) \end{pmatrix} = \mathbb{E}[\mathbf{Y}(\mathbf{0})] + \Theta \cdot \mathbf{D} + \mathbf{U}$$

where  $\mathbb{E}[\mathbf{U}|\mathbf{D}] = 0$  and, letting  $\theta_{g,t} = \mathbb{E}[Y_{g,t}(1) - Y_{g,t}(0)]$ ,

$$\mathbb{E}[\mathbf{Y}(\mathbf{0})] = \begin{pmatrix} \mathbb{E}[Y_{1,1}(0)] & \mathbb{E}[Y_{1,2}(0)] & \mathbb{E}[Y_{1,3}(0)] \\ \mathbb{E}[Y_{2,1}(0)] & \mathbb{E}[Y_{2,2}(0)] & \mathbb{E}[Y_{2,3}(0)] \\ \mathbb{E}[Y_{3,1}(0)] & \mathbb{E}[Y_{3,2}(0)] & \mathbb{E}[Y_{3,3}(0)] \end{pmatrix} \quad \Theta = \begin{pmatrix} \theta_{1,1} & \theta_{1,2} & \theta_{1,3} \\ \theta_{2,1} & \theta_{2,2} & \theta_{2,3} \\ \theta_{3,1} & \theta_{3,2} & \theta_{3,3} \end{pmatrix}$$

# Potential outcomes in panel data, $\mathbf{V}$

$$\mathbf{Y} = \mathbb{E}[\mathbf{Y}(\mathbf{0})] + \Theta \cdot \mathbf{D} + \mathbf{U} \quad \xrightarrow{(g,t) \text{ level}} \quad Y_{g,t} = E[Y_{g,t}(0)] + \theta_{g,t}D_{g,t} + U_{g,t}$$

- Linear representation of  $\mathbf{Y}$ 
  - $\mathbb{E}[Y(0)]$ : systematic component
  - $\Theta$ : treatment effects
  - $\mathbf{U}$ : idiosyncratic component
- This **latent factor** model is the key starting point of DID/SC/SDID
  - It looks like the usual linear model, so why don't we just run OLS?
- **Fundamental problem of causal inference**
  - $|G_1| \times |T_1|$  entries of  $\mathbb{E}[\mathbf{Y}(\mathbf{0})]$  are unknown (more formally, *not identified*)
  - Even if we are only interested in  $\Theta$ , we cannot "control" for  $\mathbb{E}[\mathbf{Y}(\mathbf{0})]$
  - We cannot retrieve  $\Theta$ , e.g., by linear regression, under the stated assumptions

## What do DID, SC and SDID have in common?

- Let now recast  $\Theta$  to a known object

$$\begin{aligned} ATT &= \frac{1}{|G_1|} \frac{1}{|T_1|} \sum_{g \in G_1} \sum_{t \in T_1} \mathbb{E}[Y_{g,t}(1) - Y_{g,t}(0)] \\ &= \frac{1}{|G_1|} \frac{1}{|T_1|} \sum_{g \in G_1} \sum_{t \in T_1} \Theta_{g,t} \end{aligned}$$

that is,  $ATT$  is just the average of the entries of  $\Theta$  at which  $\mathbf{D}_{g,t} = 1$

- Scope of DID/SC/SDID:

Under what assumptions on  $\mathbb{E}[\mathbf{Y}(\mathbf{0})]$  can we recover  $ATT$  from data  $(\mathbf{Y}, \mathbf{D})$ ?

## Parallel trends, I

- DID answers the last question by imposing **parallel trends**

$$\mathbb{E}[Y_{g,t}(0) - Y_{g,t-1}(0)] = \mathbb{E}[Y_{g',t}(0) - Y_{g',t-1}(0)] \quad (\text{Parallel Trends})$$

for all  $g, g' \in \{1, \dots, G\}$  and  $t \geq 2$

- Units' untreated outcomes are required to change on average by the same *amount*
  - This assumption has nothing to do with *levels*, i.e., value of  $\mathbb{E}[Y(0)_{g,t}]$
- Parallel trends has also an intuitive appeal to it
  - Without the treatment, units would have *evolved* in parallel
- Parallel trends solves the Fundamental problem in a direct way
  - Say unit 2 is treated in period 2, then the **unknown counterfactual** is **identified**

$$\mathbb{E}[Y_{2,2}(0)] = \mathbb{E}[Y_{2,1}(0)] + \mathbb{E}[Y_{1,2}(0) - Y_{1,1}(0)] = \mathbb{E}[Y_{2,1}] + \mathbb{E}[Y_{1,2} - Y_{1,1}]$$

## Parallel trends, II

- Parallel trends is equivalent to assuming a specific functional form for  $\mathbb{E}[Y_{g,t}(0)]$

$$\mathbb{E}[Y_{g,t}(0)] = \delta_g + \gamma_t$$

i.e., average untreated outcomes can be separated in a time and unit component

- This means that the latent factor model reduces to

$$Y_{g,t} = \delta_g + \gamma_t + \theta_{g,t}D_{g,t} + U_{g,t}$$

- This specification splits the unknown  $\mathbb{E}[Y_{g,t}(0)]$  into **time and unit fixed effects**

## Two-Way Fixed Effects (TWFE) regression

- Suppose we run the following least-squares regression

$$(\beta, \delta, \gamma) = \arg \min_{(b, \delta, \gamma)} \sum_{g=1}^G \sum_{t=1}^T \mathbb{E} [(Y_{g,t} - \delta_g - \gamma_t - b \cdot D_{g,t})^2]$$

- This specification is known as the **Two-Way Fixed Effects (TWFE)** regression
- It turns out that, under all the assumptions stated so far,

$$\beta = \frac{1}{|G_1| |T_1|} \sum_{g \in G_1} \sum_{t \in T_1} \mathbb{E}[Y_{g,t}(1) - Y_{g,t}(0)] = ATT$$

- As a result, one can consistently estimate ATT in this setting by running

```
reghdfe Y D, absorb(G T)
```

## Where are the differences in differences that we all love?

- The sample counterpart of TWFE is

$$(\hat{\beta}, \dots) = \arg \min_{(b, \delta, \gamma)} \sum_{g=1}^G \sum_{t=1}^T (Y_{g,t} - \delta_g - \gamma_t - b \cdot D_{g,t})^2$$

- It turns out that the TWFE estimator is exactly equal to the **diff-in-diff** estimator
  - The DID estimator compares average outcome differentials of treated and untreated units, before and after the start of the treatment

$$\hat{\beta} = \left( \frac{1}{|G_1||T_1|} \sum_{\substack{g \in G_1 \\ t \in T_1}} Y_{g,t} - \frac{1}{|G_1||T_0|} \sum_{\substack{g \in G_1 \\ t \in T_0}} Y_{g,t} \right) - \left( \frac{1}{|G_0||T_1|} \sum_{\substack{g \in G_0 \\ t \in T_1}} Y_{g,t} - \frac{1}{|G_0||T_0|} \sum_{\substack{g \in G_0 \\ t \in T_0}} Y_{g,t} \right)$$

## Caveats about TWFE

- In the basic design, TWFE is equal to the DID estimator
  - Moreover, TWFE is unbiased for ATT, i.e.,  $\mathbb{E}[\hat{\beta}] = ATT$
- This equivalence only holds in the basic design
  - With multiple cohorts, the DID estimator is not clearly defined
  - Also,  $\mathbb{E}[\hat{\beta}]$  is not guaranteed to be equal to ATT
- Beyond the basic design,  $\mathbb{E}[\hat{\beta}]$  is a weighted sum of  $\theta_{g,t}$ s
  - Weights may be **negative**
  - As such, one could get that  $\mathbb{E}[\hat{\beta}] > 0$  even if  $\theta_{g,t} < 0$
  - Policy-wise, we could pass a policy that makes everyone worse off
  - More on these issues in Chapter 5 of *Credible Answers to Hard Questions*

# A factor model approach, I

- Parallel trends assumption is often not verified
  - Treated and untreated groups are generally on different outcome trajectories
  - People join a training program *because* they experience a decline in their income
- The additive structure implied by parallel trends is not necessary to get ATT
- The baseline motivation for a **synthetic control** approach is more flexible form for the systematic component of the latent factor model

$$\mathbb{E}[Y_{g,t}(0)] = \Delta'_g \Gamma_t = \sum_{j=1}^k \Delta_{g,j} \Gamma_{t,j}$$

where  $\Delta_g$  (**unit factor**) and  $\Gamma_t$  (**time factor**) are vectors in  $\mathbb{R}^k$

- E.g.,  $\Delta_g = (\delta_{g,1}, \dots, \delta_{g,k})$  and  $\Gamma_t = (\gamma_{t,1}, \dots, \gamma_{t,k})$

## A factor model approach, II

- This specification actually embeds parallel trends
  - Say  $\Delta_g = (\delta_g, 1)$  and  $\Gamma_t = (1, \gamma_t)$ , then  $\mathbb{E}[Y_{g,t}(0)] = \delta_g + \gamma_t$
- It also allows for non-parallel trends

$$\mathbb{E}[Y_{g,t}(0) - Y_{g,t-1}(0)] - \mathbb{E}[Y_{\tilde{g},t}(0) - Y_{\tilde{g},t-1}(0)] = (\Delta_g - \Delta_{\tilde{g}})'(\Gamma_t - \Gamma_{t-1})$$

where the right-hand side need not to be 0

- SC was originally proposed for settings where only 1 or few units were treated
  - To enforce this, let  $g_1 = |G_0| + 1$  and  $t_1 = |T_0| + 1$

## A factor model approach, III

- The ATT reduces to

$$ATT = \mathbb{E}[Y_{g_1, t_1}(1) - Y_{g_1, t_1}(0)]$$

- The Fundamental problem strikes back:  $\mathbb{E}[Y_{g_1, t_1}(0)]$  is not observed
- Say, however, that there are weights  $\omega$  in  $[0, 1]$  such that

$$\sum_{g \in G_0} \omega_g \Delta_g = \Delta_{g_1} \quad \sum_{g \in G_0} \omega_g = 1$$

- The unit factor of the treated unit lies in the **convex hull** of control units' factors
- A **synthetic** control unit can be formed by convex combination of untreated units

$$Y_t^* = \sum_{g \in G_0} \omega_g Y_{g, t}$$

## A factor model approach, IV

$$(1) \mathbb{E}[Y_{g,t}(0)] = \Delta'_g \Gamma_t \quad (2) \sum_{g \in G_0} \omega_g \Delta_g = \Delta_{g_1} \quad (3) Y_t^* = \sum_{g \in G_0} \omega_g Y_{g,t}$$

- Consider the cross-sectional difference between  $g_1$  and the SC in the post period

$$\mathbb{E}[Y_{g_1,t_1}] - \mathbb{E}[Y_t^*] = ATT + \mathbb{E}[Y_{g_1,t_1}(0)] - \sum_{g \in G_0} \omega_g \mathbb{E}[Y_{g,t_1}(0)] \quad (3)$$

$$= ATT + \Delta'_{g_1} \Gamma_{t_1} - \sum_{g \in G_0} \omega_g \Delta'_g \Gamma_{t_1} \quad (1)$$

$$= ATT + \left( \Delta_{g_1} - \sum_{g \in G_0} \omega_g \Delta_g \right)' \Gamma_{t_1}$$

$$= ATT \quad (2)$$

- SC identifies the ATT under the factor model (1) and convex hull (2) assumptions

## Implementation of SC, I

- The simplest way to compute the weights is to project outcomes in the pre period

$$\hat{\omega} = \arg \min_{\omega \in [0,1]^{G_0}} \sum_{t \in T_0} \left( Y_{g_1,t} - \sum_{g \in G_0} \omega_g Y_{g,t} \right)^2 \quad \text{s.t.} \quad \sum_{g \in G_0} \omega_g = 1$$

- The SC estimator is

$$\hat{\tau}^{SC} = Y_{g_1,t_1} - \sum_{g \in G_0} \hat{\omega}_g Y_{g,t_1}$$

- In this way, the validity of SC hinges on the condition that

$$\sum_{g \in G_0} \hat{\omega}_g Y_{g,t} \approx Y_{g_1,t} \text{ for } t \in T_0 \quad \text{then} \quad \sum_{g \in G_0} \hat{\omega}_g \Delta_g \approx \Delta_{g_1}$$

that is, estimated weights well approximate those that would give us the ATT

## Implementation of SC, II

- In a more general setting, suppose that  $G_1$  and  $T_1$  are small, with  $|G_1|, |T_1| > 1$
- Define the weights by balancing the average treated units' outcomes

$$\hat{\omega} = \arg \min_{\omega \in [0,1]^{G_0}} \sum_{t \in T_0} \left( \frac{1}{|G_1|} \sum_{g \in G_1} Y_{g,t} - \sum_{g \in G_0} \omega_g Y_{g,t} \right)^2 \quad \text{s.t.} \quad \sum_{g \in G_0} \omega_g = 1$$

- The SC estimator is a cross-sectional difference, so we can define for  $t \in T_1$

$$\hat{\tau}_t^{SC} = \frac{1}{|G_1|} \sum_{g \in G_1} Y_{g,t} - \sum_{g \in G_0} \hat{\omega}_g Y_{g,t}$$

- Under regularity conditions, this is an (asymptotically) unbiased estimator for

$$ATT_t = \frac{1}{|G_1|} \sum_{g \in G_1} \mathbb{E}[Y_{g,t}(1) - Y_{g,t}(0)]$$

## Implementation of SC, III - Aggregated estimator

- Consider the following weighted fixed regression

$$(\hat{\tau}^{SC}, \cdot) = \arg \min_{b, \gamma} \sum_{g=1}^G \sum_{t=1}^T (Y_{g,t} - \gamma_t - b \cdot D_{g,t})^2 \hat{\omega}_g^{SC}$$

where

$$\hat{\omega}_g^{SC} = \begin{cases} \hat{\omega}_g & \text{if } g \in G_0 \\ \frac{1}{|G_1|} & \text{if } g \in G_1 \end{cases}$$

- The weighted fixed effects estimator is equal to the average of all SC estimators

$$\hat{\tau}^{SC} = \frac{1}{|T_1|} \sum_{t \in T_1} \left( \frac{1}{|G_1|} \sum_{g \in G_1} Y_{g,t} - \sum_{g \in G_0} \hat{\omega}_g Y_{g,t} \right) = \frac{1}{|T_1|} \sum_{t \in T_1} \hat{\tau}_t^{SC}$$

## Abadie et al. (2010)

- Context: California's "Proposition 99" tobacco control policy (1989)
- California increased sales tax for cigarettes by 25 cents
- Find significant reduction in per-capita cigarette sales
- ! This paper is the standard example of SC
- Data: state-by-year panel
  - Years: 1970-2000
  - 38 control states, only 1 treated
- We use per-capita cigarette sales as the relevant outcome

## Implementation using `sdid`

- The `sdid` package (Stata, R) can also be used to estimate DID and SC
- Install the package

```
ssc install sdid, replace
```

- Load the data

```
clear  
webuse set www.damianclarke.net/stata/  
webuse prop99_example.dta, clear
```

- SDID syntax

```
sdid Y G T D [if] [in] [weight] [, options]
```

## DID and SC - Mind the line breaks when copying-and-pasting!

- DID - Graph comparing  $Y_{g_1,t}$  vs  $\frac{1}{|G_0|} \sum_{g \in G_0} Y_{g,t}$

```
sdid packspercapita state year treated, method(did) vce(noinference)
graph
```

- SC - Graph comparing  $Y_{g_1,t}$  vs  $\sum_{g \in G_0} \hat{\omega}_g Y_{g,t}$

```
sdid packspercapita state year treated, method(sc) vce(noinference)
graph
```

- SC - Graph comparing  $\frac{1}{|T_1|} \sum_{t \in T_1} Y_{g_1,t}$  vs  $\frac{1}{|T_1|} \sum_{t \in T_1} Y_{g,t}$ , size =  $\hat{\omega}_g$

```
sdid packspercapita state year treated, method(sc) vce(noinference)
graph g1on
```

## Next session

- Synthetic Differences-in-Differences
  - Estimation & TWFE representation
  - Unit and time synthetic weights
  - Double-robustness
- Staggered adoption design & SDID
  - Cohort-level SDID estimators
- Event study disaggregation
  - Event-time level SDID estimators

Thanks!